

# Guide to Maths for Geographers



**GCSE (9-1) Geography A and B**

**Pearson Edexcel Level 1/Level 2 GCSE (9-1) in Geography A (1GA0) and B (1GB0)**

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# GCSE Guide to Maths for Geographers

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## Introduction

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This guide to maths for geographers outlines the content that students will have covered in their maths lessons throughout KS3 and KS4. You can use this guide to help you understand how different areas are approached in maths, and therefore support your teaching of mathematical content in geography lessons.

The content is split into distinct mathematical concepts. Each chapter takes you through the terminology used in that area, as well as examples taken from Pearson maths textbooks to show you the methods students should be familiar with when solving mathematical problems.

Sections which are highlighted green have a particular connection or reference to geography and may include geographical examples.

Sections which are highlighted blue refer to inconsistencies and common errors.

# 1. Statistical graphs, charts and tables: data, bar charts, frequency tables and diagrams, pie charts, histograms

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## 1.1 Data

### Demand

All students learn the difference between discrete and continuous data in KS3. They also come across categorical data.

### Terminology

Data is either qualitative (descriptive) or quantitative (numerical), as well as either discrete (only certain values according to context) or continuous (any value).

**Inconsistency:** in geography discrete data may be called discontinuous data.

- Qualitative data is information that describes something in more subjective terms, e.g. what people think about the town they live in.
- Quantitative data is data that has a numerical value and can be measured precisely, e.g. the number of cars parked on a road.
- Discrete data can only take certain values, e.g. whole numbers, or shoe sizes.
- Continuous data is measured, e.g. length, time, and can take any value.
- Categorical data is where there is no numerical value, but data can still be sorted into groups, e.g. preferences from an interview.

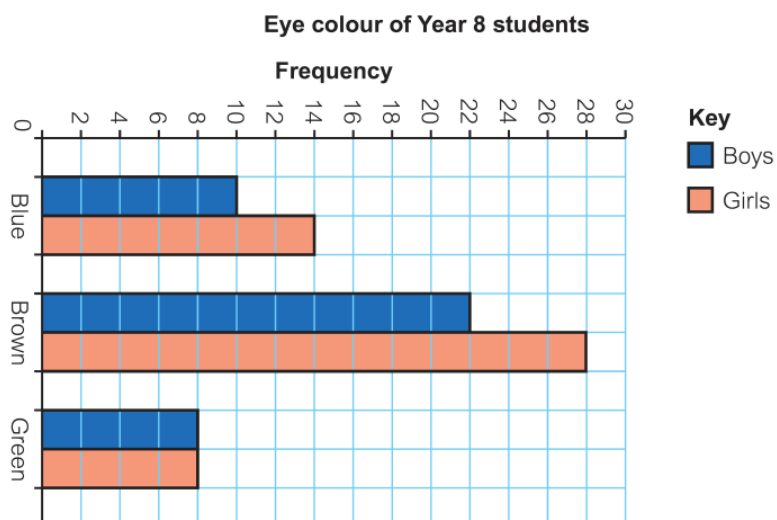
## 1.2 Bar charts

### Demand

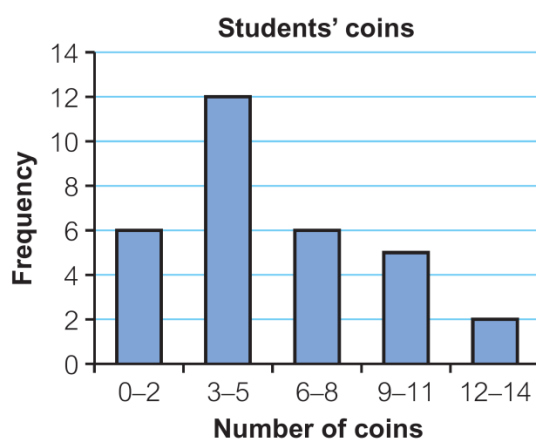
All KS3 student learn to draw and interpret bar charts for discrete and continuous data. may need help with interpreting scales on axes given in, for example, thousands (i.e. 2.2 thousand = 2200) or millions.

### Approach

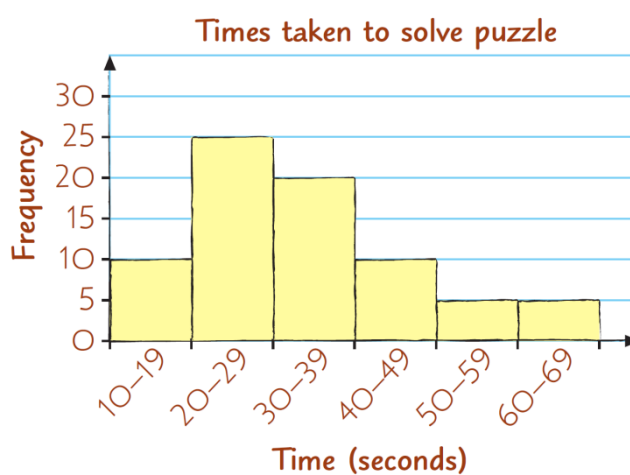
- Can show qualitative or quantitative, discrete or continuous data.
- One axis is usually labelled 'Number of ...' or shows frequency.
- Frequency is usually shown on the vertical axis (but can be on the horizontal axis with the bars in the chart shown horizontally).
- Bars should be equal in width.
- For discrete and qualitative data there are gaps between the bars.
- A bar-line graph, for discrete and qualitative data, uses lines instead of bars. It can be used to save time drawing the bars.
- For continuous data there are no gaps between the bars.
- In questions on interpreting proportions from bar charts, ask for the fraction or percentage of students with brown eyes, not 'the proportion of students'.



**Figure 1** Horizontal bar chart (discrete, qualitative data)



**Figure 2** Bar chart (discrete, quantitative data): gaps between the bars



**Figure 3** Bar chart (continuous, quantitative data): no gaps between the bars



**Figure 4** Bar-line chart (discrete, quantitative data)

## 1.3 Frequency table

### Demand

All KS3 students learn to draw and interpret frequency tables for discrete and continuous data.

### Approach

- A table of data that shows the number of items, or frequency of each data value or each data group.
- Data can also be grouped. For discrete data use groups such as 0–5, 6–10, etc. For continuous data use groups such as  $0 \leq t < 10$ ,  $10 \leq t < 20$ . The groups must not overlap.
- In maths, students learn that it is best to group numerical data into a maximum of 6 groups. If you need them to group data differently, tell them how many groups of equal width to group it into.

Shoe size	Frequency
3	3
4	5
5	7
6	10
7	10
8	6
9	1

**Figure 5** Frequency table (ungrouped discrete, quantitative data)

Geography mark	0–10	11–20	21–30	31–40	41–50
Frequency	4	13	17	19	7

**Figure 6** Frequency table (grouped discrete, quantitative data)

Distance ( $d$ metres)	Frequency
$10 \leq d < 20$	2
$20 \leq d < 30$	6
$30 \leq d < 40$	15
$40 \leq d < 50$	20
$50 \leq d < 60$	4

**Figure 7** Frequency table (grouped continuous data)

## 1.4 Frequency diagram

### Demand

All KS3 students learn to draw and interpret bar charts for discrete and continuous data. Lower ability maths students (KS3 and KS4) may need help with interpreting scales on axes given in, for example, thousands (i.e. 2.2 thousand = 2200) or millions.

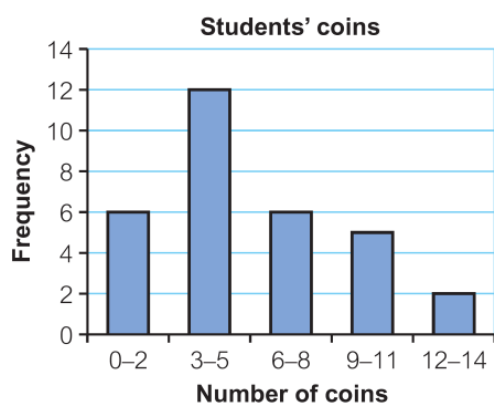
### Approach

#### Inconsistency

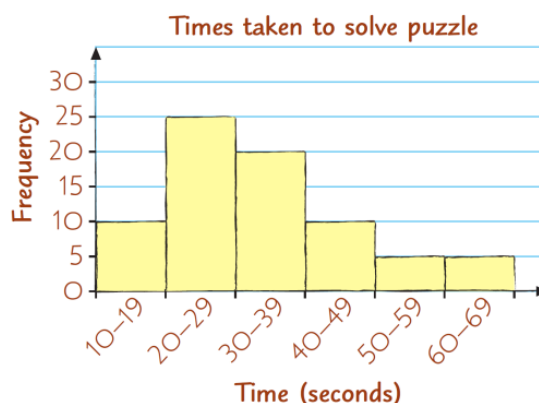
In KS3 maths, the name 'Frequency diagram' is not used.

- Frequency diagram is another name for a bar chart where the vertical axis is labelled Frequency.
- They can be used to show discrete or continuous data.

For example, these two bar charts could also be called frequency diagrams:



**Figure 8** Frequency diagram (discrete, quantitative data)



**Figure 9** Frequency diagram (continuous quantitative data)

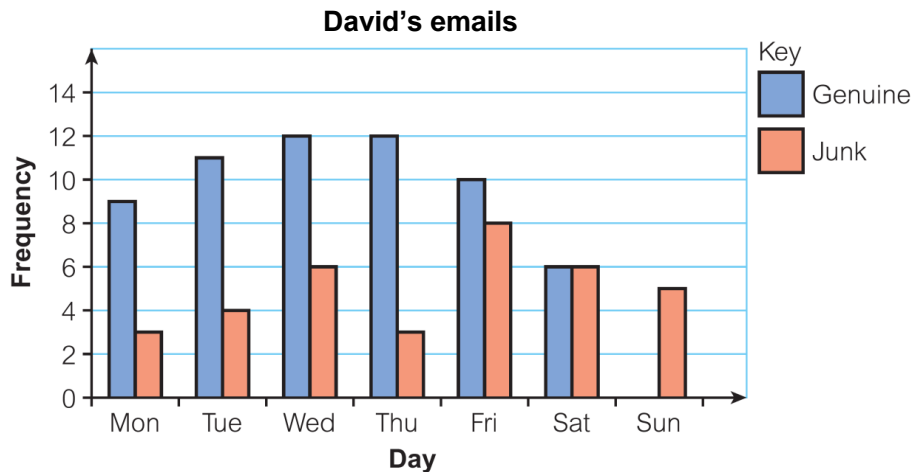
## 1.5 Comparative bar chart

### Demand

All KS3 students learn to draw and interpret comparative bar charts.

### Approach

- Compares two or more sets of data.
- Uses different coloured bars for each set of data.
- Needs a key to show what each colour bar represents.



**Figure 10** Comparative bar chart (discrete, qualitative data)

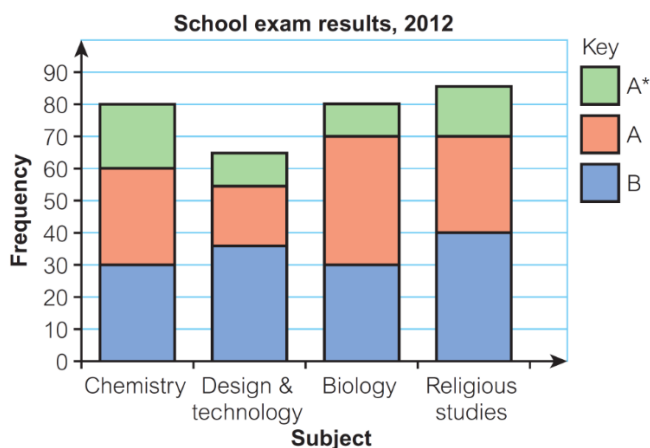
## 1.6 Compound bar chart

### Demand

All KS3 students learn to draw and interpret compound bar charts.

### Approach

- Combines different sets of data in one bar.
- Needs a key to show what each colour section represents.
- In questions on interpreting proportions in compound bar charts, ask for the fraction or percentage of chemistry students getting A\*, not 'the proportion of students'.



**Figure 11** Compound bar chart (discrete, quantitative data)



## 1.7 Histogram

### Demand

The bar charts students draw in KS3 for grouped continuous data could be called histograms, although students do not meet histograms until KS4

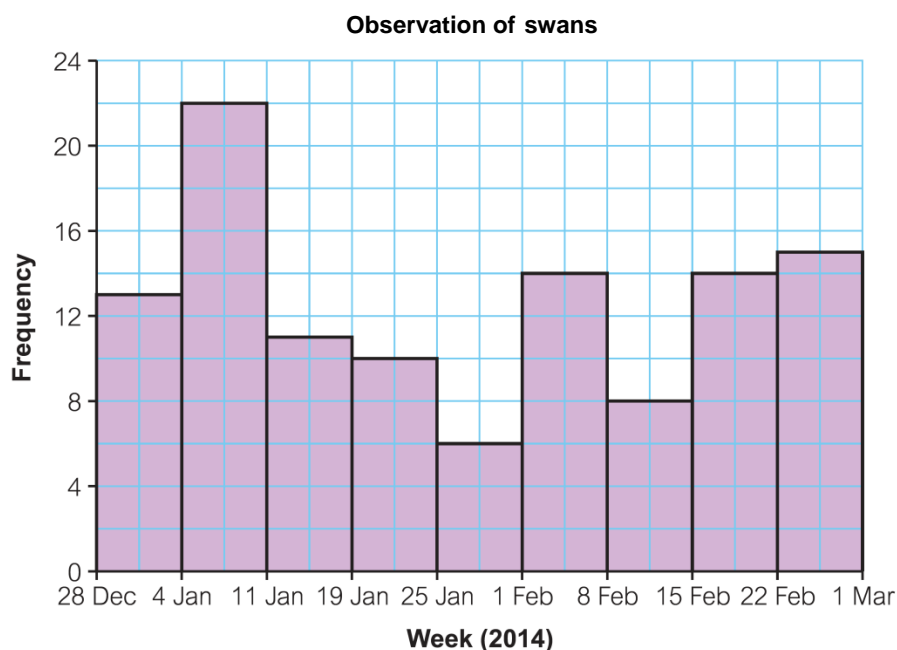
**Only Higher Tier GCSE students** study histograms with unequal width bars/groups, where frequency density is plotted on the vertical axis.

### Approach

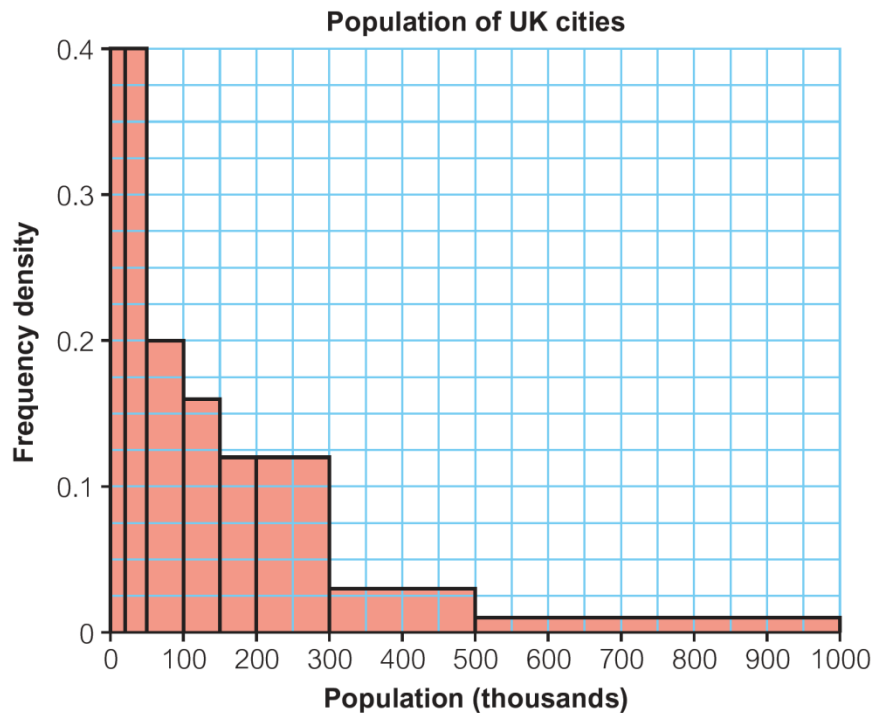
- Can be drawn for grouped continuous data where groups/bars are of equal width.
- No gaps between the bars.
- If groups/bars are of unequal width, the vertical axis is labelled frequency density, which is calculated as:

$$\frac{\text{number in group}}{\text{group width}}$$

- The area of the bar is proportional to the number of items it represents (frequency).



**Figure 12** Histogram with equal width bars/groups



**Figure 13** Histogram with unequal width bars/groups: used when the data is grouped into classes of unequal width

## Drawing a histogram

### Key point 12

In a **histogram** the area of the bar represents the frequency. The height of each bar is the frequency density.

$$\text{Frequency density} = \frac{\text{frequency}}{\text{class width}}$$

### Example 4

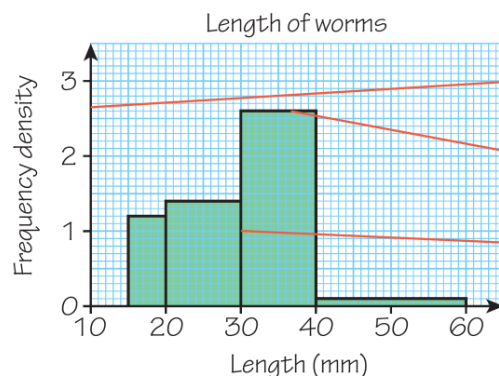
The lengths of 48 worms are recorded in this table.

Length, $x$ (mm)	$15 < x \leq 20$	$20 < x \leq 30$	$30 < x \leq 40$	$40 < x \leq 60$
Frequency	6	14	26	2

Draw a histogram to display this data.

$$6 \div 5 = 1.2, 14 \div 10 = 1.4, 26 \div 10 = 2.6, 2 \div 20 = 0.1$$

Work out the frequency density for each class



Label the  $y$ -axis 'Frequency density'.

The height of each bar is the frequency density for each class.

Draw the bars with no gaps between them.

Source: Edexcel GCSE (9-1) Mathematics Higher student book

# 1.8 Pie charts

## Demand

All KS3 students should learn how to construct a simple pie chart.

Lower ability maths students (KS3 and KS4) will probably struggle with working out the angles as they will not have learned how to calculate percentages or fractions that are not nice round numbers.

All KS4 students should know how to draw and interpret pie charts.

## Terminology

- A pie chart is a circle divided into **sectors**.  
NB: a 'slice' of a pie is a sector, not a **segment**.
- The angle of each sector is proportional to the number of items in that category
- Shows proportions of a set of data, e.g. fraction or percentage of waste recycled
- May need a key.

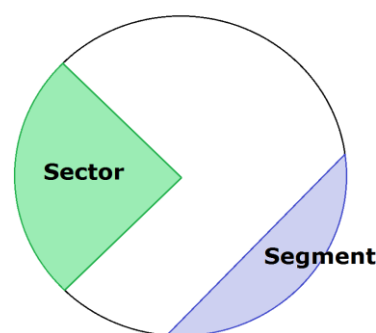


Figure 14 Pie chart terminology

## Approach

### Drawing a pie chart

#### Example 5

The table shows the match results of a football team. Draw a pie chart to represent the data.

Result	Won	Drawn	Lost
Frequency	28	12	20

Total number of games =  $28 + 12 + 20 = 60$

$\div 60$   $60 \text{ games} : 360^\circ$   
 $1 \text{ game} : 6^\circ$

The total number of games is the total frequency.

1 game =  $360 \div 60 = 6^\circ$

Work out the angle for one game.

Won:  $28 \times 6^\circ = 168^\circ$

Drawn:  $12 \times 6^\circ = 72^\circ$

Work out the angle for each result.

Lost:  $20 \times 6^\circ = 120^\circ$

Check:  $168 + 72 + 120 = 360$

Check that your angles total  $360^\circ$ .

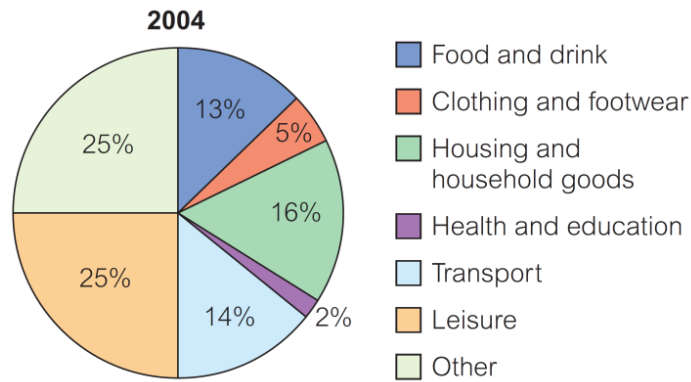
Team results



Draw the pie chart. Give it a title and label each section, or make a key.

Source: Edexcel GCSE (9-1) Mathematics Foundation student book

In questions asking students to interpret a pie chart, ask for the 'fraction' or 'percentage' who learn Geography, not the 'proportion' who learn Geography.



**Figure 15** Pie chart showing average household spending (percentages) in 2004

## 1.9 Drawing graphs and charts to display data

### Demand

Choosing a suitable graph or chart to draw to display data is quite a high level skill in maths.

Lower ability maths students (KS3 and KS4) may therefore need some guidance on the type of chart to draw for given sets of data. They may also need help with choosing suitable scales for axes.

Geography students should think carefully about the number of categories that can be used so that the graphs make sense, and why they might need to use an alternative method sometimes.

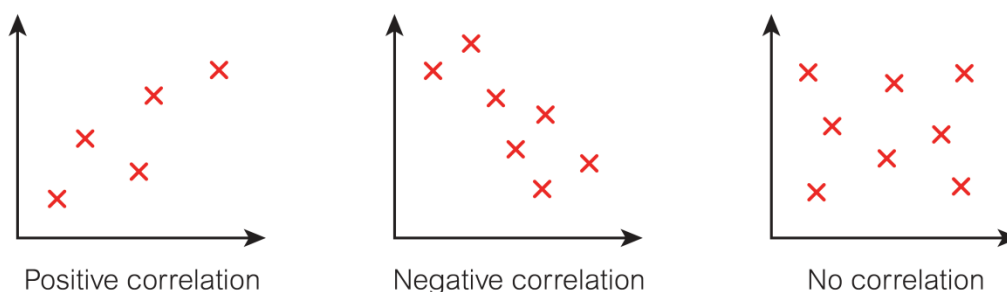
## 2. Graphs including lines of best fit, proportionality, gradients, relationships and correlations

### 2.1 Scatter graphs

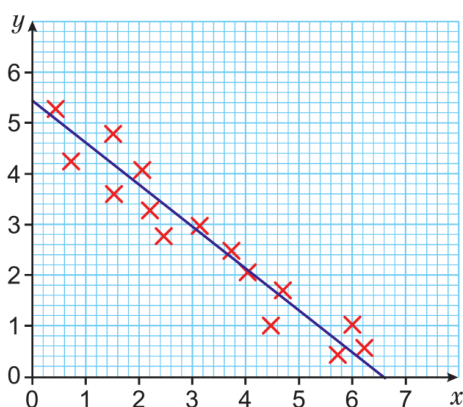
#### Terminology

**Inconsistency:** In maths these are usually called scatter graphs, not scatter diagrams or scatter plots. In geography we may, unknowingly, mix up the terminology and confuse.

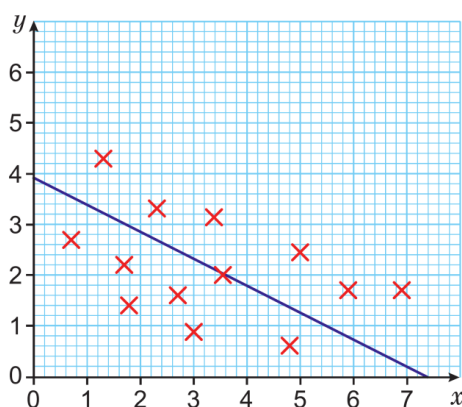
- A scatter graph plots two sets of data on the same graph to see if there is a relationship or correlation between them.
- Scatter graphs can show positive, negative or no correlation.



- Correlation is when two sets of data are linked. For example, when one value increases as the other decreases, or when one value decreases as the other increases.
- In maths, points on scatter graphs are plotted with crosses.
- The line of best fit follows the shape of the data and has roughly the same number of crosses above and below the line. There may also be crosses on the line.



**Figure 16** Strong correlation

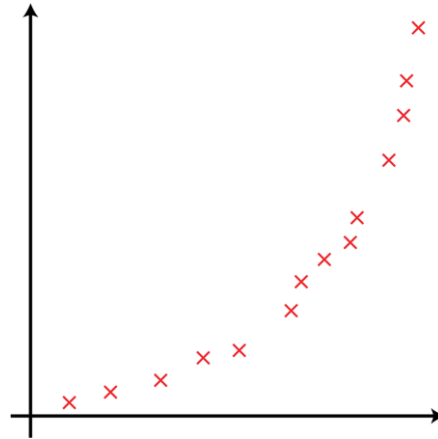


**Figure 17** Weak correlation

**Inconsistency:** In maths, when interpreting a scatter graph an acceptable answer is 'shows positive correlation', unless the question explicitly asks for this to be explained in context.

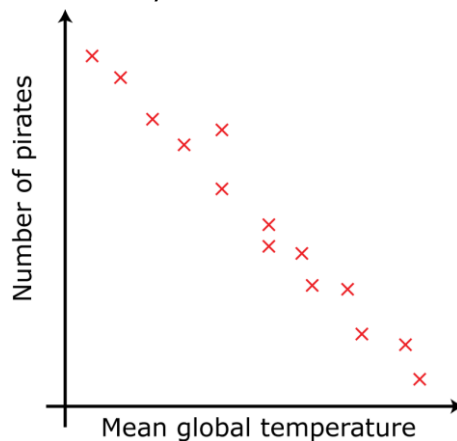
We should use the same language in geography, but also comment on the geographical meaning or relevance of such a relationship.

- The line of best fit shows a relationship between two sets of data.
- When the points on a scatter graph are on or close to a straight line:
  - there is strong correlation between the variables
  - there could be a linear relationship between the variables, e.g.  $y = mx$  or  $y = mx + c$ . The equation of the line of best fit describes this relationship.
- When the points on a scatter graph are not close to a straight line there may be another relationship between the variables.

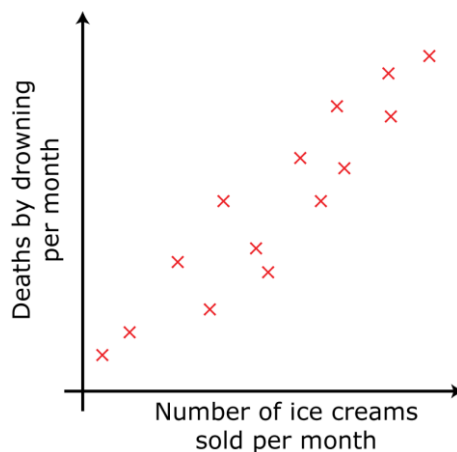


**Figure 18** Scatter graph showing a possible non-linear relationship

- Correlation does not imply causation. Sometimes there may be another factor that affects both variables, or there may be no connection between them at all.



There is a negative correlation between number of pirates and mean global temperature, but it is unlikely that one causes the other!



There is positive correlation between number of ice-creams sold and death by drowning, but it is unlikely that one causes the other (i.e. directly causal). A more likely explanation is a third factor – temperature. On hot days more people buy ice creams and more people swim, leading to increased numbers of drownings.

Many areas of geography do not show clear causal linkages and so factors beyond the relationship shown in the graph must be considered.

## Approach

### Drawing scatter graphs

Lower ability maths students would not be expected to know which variables to put on which axis for a scatter graph. They may need help with deciding which is the independent variable, and a reminder that this goes on the horizontal axis.

### Drawing a line of best fit

Place your ruler on the graph, on its edge. Move the ruler until it is following the shape of the data, with roughly the same number of points above or below it. Ignore any points on the line.

#### Common error

Students often try to make their lines of best fit go through (0, 0). A line of best fit does not necessarily pass through the origin. It should stop at the first or last plot point or cross.

In geography we often don't have data (especially for fieldwork) that goes to zero, so students need to avoid manipulating their lines of best fit to go through (0, 0).

## 2.2 Interpreting scatter graphs

### Demand

All KS3 students should have learned to interpret scatter graphs. They will have learned about correlation and causation in KS3 maths.

Lower ability students at GCSE would not be expected to know which variables to put on which axis for a scatter graph.

Students are not expected to find equation of a curve of best fit in GCSE maths.

At GCSE Higher level students would be expected to state that there is a possible non-linear relationship if the points on a scatter graph closely follow a smooth curve.

#### Common error

Students often find interpreting scatter graphs difficult as they do not know how to put into words what the graph shows, so it is good to give them examples of graphical interpretations, or at least sentences to copy and complete, such as:

- the steeper the gradient the \_\_\_\_\_ the river's energy.

You can also use statements such as:

- the greater the \_\_\_\_\_, the longer the \_\_\_\_\_.

## 2.3 Line graphs

### Demand

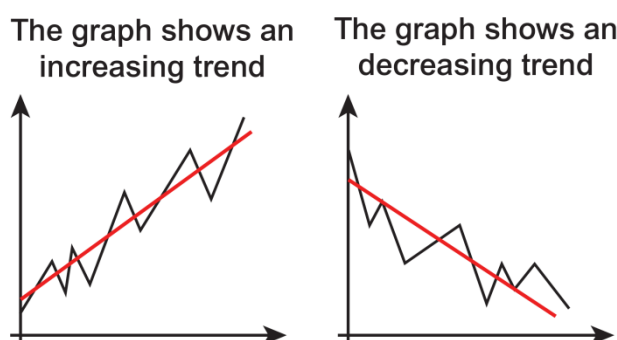
Lower ability students at KS3 would not be expected to know which variables to put on which axis for a scatter graph. They may need help with deciding which is the independent variable, and a reminder that this goes on the horizontal axis.

In geography the independent variable is often time.

### Terminology

- In maths, a line graph that shows how a variable changes over time (i.e. with time on the horizontal axis) is often called a time-series graph.
- Line graphs can show trends in data. The trend is the general direction of change, ignoring individual ups and downs.

In geography we are used to seeing climate change graphs, surface temperatures, with trend lines, or population predictions, for example. This website has climate related graphs (from the IPCC) which show clear trends: <http://www.carbonbrief.org/ipcc-six-graphs-that-explain-how-the-climate-is-changing>



**Figure 19** Examples of graphs showing increasing and decreasing trends

### 2.3.1 Drawing line graphs

#### Demand

Lower ability students at KS3 would not be expected to know which variables to put on which axis for a line graph. They may need help with deciding which is the independent variable, and a reminder that this goes on the horizontal axis.

Choosing a suitable graph or chart to draw to display data is quite a high level skill in maths.

Lower ability students in KS3 and Foundation GCSE maths students may need some guidance on the type of chart to draw for given sets of data.

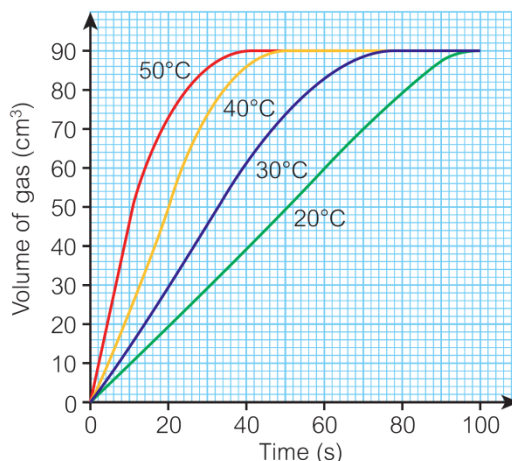
Lower ability maths students may also need help with choosing suitable scales for axes.

#### Approach

- In maths, students draw graphs on squared or graph paper.
- They plot points with crosses (x).
- They join points with straight lines or a smooth curve – the question needs to tell them which to use.
- All graphs should have labels on the axes and a title.



- When more than one data set is shown, the lines could be different to show this clearly, e.g. one solid and one dashed line. The graph will need a key to explain solid/dashes. Alternatively, colour-code the lines according to category.



**Figure 20** Line graph showing more than one data set.

### 2.3.2 Interpreting line graphs

#### Demand

All KS3 students should learn to interpret line graphs.

Lower ability maths students (KS3 and KS4) may need help with interpreting scales on axes given in, for example, thousands (i.e. 2.2 thousand = 2200) or millions.

At KS4 all students will have limited experience of interpreting real life graphs that dip below zero.

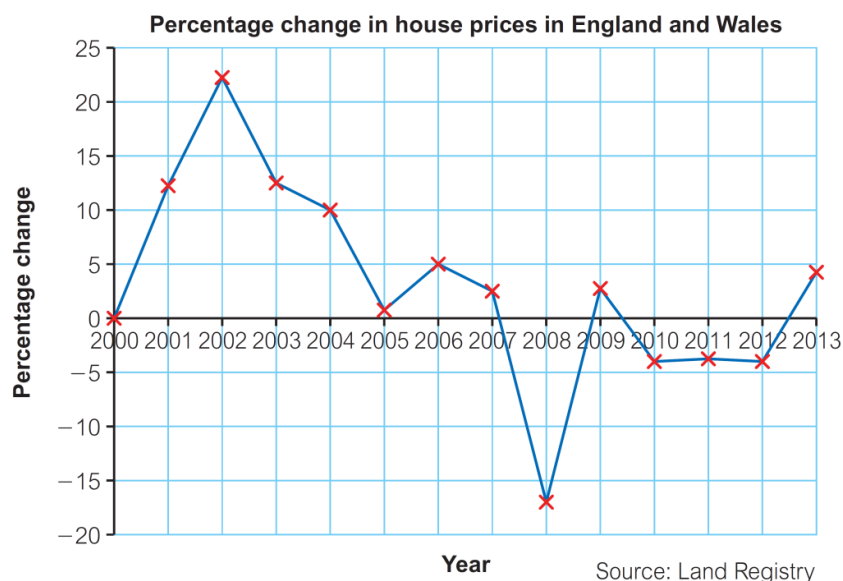
Only higher ability maths students are likely to have seen graphs with two different vertical scales to read from.

#### Approach

Students interpret 'real life' graphs in maths, in a variety of contexts.

They may be less familiar with:

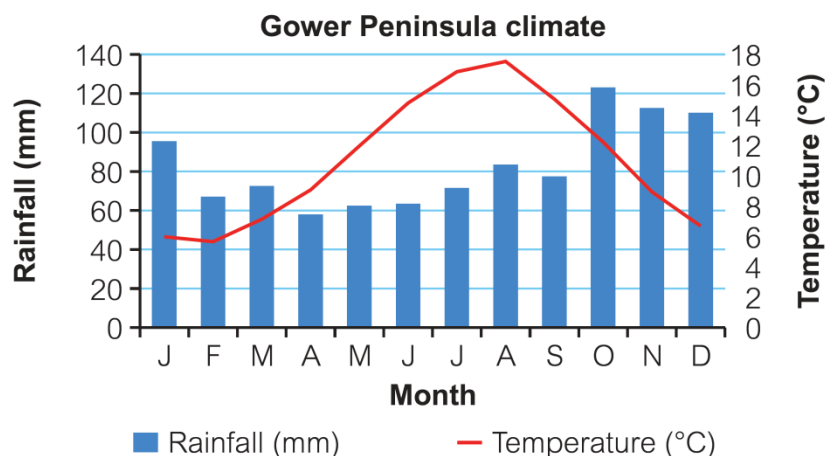
- graphs showing negative values such as the one below.



**Figure 21** Graph showing negative values

- two types of graph on one set of axes, and two different vertical axes for the same graph.

In geography an example of this is a climate graph. It is normal to show temperature a continuous line and rainfall using discrete bars (as its monthly average totals). This kind of data might be relevant to weather and climate change.



**Figure 22** Graph showing two types of graph on one set of axes, and two different vertical axes

Ensure the terms used in the question match the labels on the graph.

## 2.4 Recognising proportional relationships from graphs

### Demand

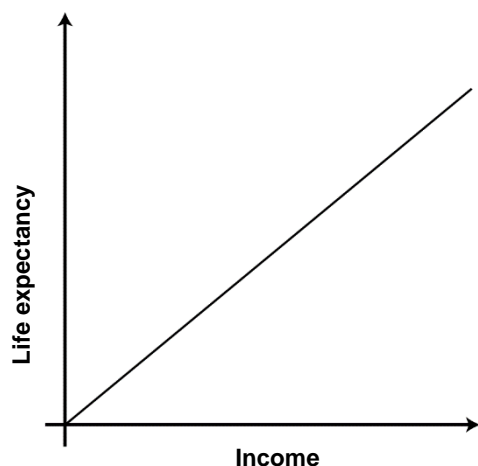
In year 9, the majority of maths students should know that a straight line graph shows the two variables are in direct proportion (please use direct proportion, not just proportion). Only top sets maths students will have met graphs showing inverse proportion.

In KS4 all students should learn that the origin is the point (0, 0).

In KS4 all students will meet graphs showing inverse proportion.

### Terminology

- A straight line graph through the origin (0, 0) shows that the two variables are in **direct proportion**.  
When one variable doubles, so does the other. When one halves, so does the other. The relationship is of the form  $y = mx$ , where  $m$  is the gradient of the graph.
- A straight line graph not through the origin shows a **linear relationship**.  
The relationship is of the form  $y = mx + c$ , where  $m$  is the gradient of the graph and  $c$  is the  $y$ -intercept (where the graph crosses the  $y$ -axis).



**Figure 23** Graph showing two variables in direct proportion to each other

The graph shows a directly theoretical proportional relationship between a country's income and life expectancy. The gradient is  $m \approx 10$ .

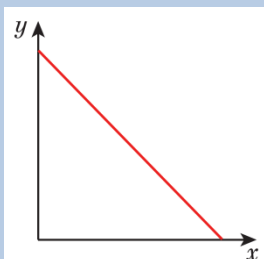
You can find more relationships such as this on the Gapminder website

<http://www.gapminder.org/world/>

This type of data is relevant to development, inequality and globalisation topics, amongst others.

#### Common error

A straight line graph with negative gradient shows a linear relationship – not inverse proportion. This is a fairly common misconception.



### 3. Fractions, percentages, ratios

#### 3.1 Fractions

##### Demand

All KS3 students learn how to add, subtract, multiply and divide decimals and find a fraction of a quantity.

They also learn how to convert fractions to decimals and vice versa, and to use and interpret recurring decimal notation.

Lower ability KS3 students will not learn to find the reciprocal of a fraction. All students will learn this in GCSE maths.

##### Approach

##### Convert a fraction to a decimal

Divide the top number by the bottom number.

For example:  $\frac{3}{8} = 0.375$

$$\frac{12}{50} = 0.24$$

##### Convert a decimal to a fraction

##### Worked example

Write 0.32 as a fraction in its simplest form.

$$0.32 = \frac{32}{100} \quad \begin{array}{c} \div 4 \\ \frac{32}{100} = \frac{8}{25} \\ \div 4 \end{array}$$

...	H	T	U	.	$\frac{1}{10}$	$\frac{1}{100}$	...
			0	.	3	2	

0.32 is the same as  $\frac{32}{100}$

Source: KS3 Maths Progress

##### Calculate a fraction of a quantity

##### Example 3

Work out  $\frac{3}{5}$  of 40. — In mathematics, 'of' means multiply.

$$\frac{1}{5} \text{ of } 40 = \frac{1}{5} \times 40 = 40 \div 5 = 8$$

$$\frac{3}{5} \text{ of } 40 = 3 \times 8 = 24$$

Multiply by 3 to find  $\frac{3}{5}$

Source: Edexcel GCSE (9-1) Mathematics

##### Reciprocals

The reciprocal of a number is 1 divided by the number.

The reciprocal of 4 is  $\frac{1}{4}$  or 0.25

The reciprocal of  $\frac{3}{4}$  is  $\frac{4}{3}$ .

The reciprocal of  $\frac{1}{2}$  is  $\frac{2}{1} = 2$ .

## Terminology

- Write fractions on two lines, i.e.  $\frac{1}{1000}$  not 1/1000 on one line.
- Avoid 'cancelling' – instead, write fractions in their simplest form.
- In a fraction, the horizontal line means 'divide'. So  $\frac{3}{5}$  means  $3 \div 5$ . Understanding this helps students remember how to convert fractions to decimals.
- A 'dot' over a decimal value shows the number recurs, e.g.  $0.\dot{6}$  means 0.66666...
- A dot over two decimal values shows the numbers between the dots recur, e.g.  $0.\dot{1}5$  means 0.151515... and  $0.2\dot{4}75\dot{1}$  means 0.247514751...
- The reciprocal of a number is  $1 \div$  the number. For fractions, this means the reciprocal of  $\frac{a}{b} = 1 \div \frac{a}{b} = 1 \times \frac{b}{a} = \frac{b}{a}$ .
- Dividing by a fraction is the same as multiplying by its reciprocal.

## 3.2 Percentages

### Demand

In KS3, students in lower maths sets will only have written one number as a percentage of another where the larger number is a multiple or factor of 100, so they may find this difficult. They would not be expected to do this calculation in maths without a calculator.

When students can be expected to use a multiplier.

Finding original amount calculations.

### Percentage change

Foundation level students learn this in year 11.

### Approach

#### Convert a percentage to a fraction

##### Worked example

Write 20% as a fraction.

$$20\% = \frac{20}{100} = \frac{2}{10} = \frac{1}{5}$$

$\xrightarrow{\div 10} \quad \xrightarrow{\div 2}$   
 $\xleftarrow{\div 10} \quad \xleftarrow{\div 2}$

First write 20% as a fraction of 100.

Then simplify the fraction by dividing the numerator and denominator by the same number. Keep doing this until the fraction is in its simplest form.

Source: *KS3 Maths Progress*

#### Convert a percentage to a decimal

##### Worked example

Write 35% as a decimal.

$$35\% = \frac{35}{100} = 0.35$$

Write 35% as a fraction out of 100. Then divide 35 by 100 to write it as a decimal.

Source: *KS3 Maths Progress*

**Convert a fraction to a percentage**

Convert the fraction to a decimal, then convert the decimal to a percentage.

For example:  $\frac{34}{80} = 0.425 = 42.5\%$

Students can input  $\frac{34}{80}$  as a fraction into a scientific calculator and press = (or the S-D button on some calculators) to get the equivalent decimal.

**Write one number as a percentage of another**

Write as a fraction, then convert to a percentage.

For example, in a class of 28 students, 13 are boys. What percentage are boys?

$$\frac{13}{28} = 0.4642... = 46.4\% \text{ (1 dp)}$$

Without a calculator:

$$\text{Percentage of charity shops} = \frac{\text{number of charity shops}}{\text{total number of shops}} \times 100$$

$$\begin{aligned} &= \frac{24}{40} \times \frac{20}{100} \\ &= \frac{3}{5} \times \frac{1}{5} \\ &= \frac{3}{25} \\ &= 60\% \end{aligned}$$

**Calculating a percentage of an amount**

50% is the same as  $\frac{1}{2}$ , so to find 50% divide by 2.

10% is the same as  $\frac{1}{10}$ , so to find 10% divide by 10.

To calculate 30% mentally, find 10% and multiply by 3.

To calculate 5% mentally, find 10% and halve it.

**Calculating percentages using a calculator*****Input the percentage as a fraction***

For example, to calculate 30% of 20 m, input  $\frac{30}{100} \times 20$  and press = to get 6 m.

***Input the percentage using a decimal multiplier***

$$65\% = 0.65$$

So to calculate 65% of 80, input  $0.65 \times 80$  and press = to get 52.

**Percentage increase/decrease*****Work out the increase and add it on/subtract it*****Examples**

To increase 45 by 20%

$$20\% \text{ of } 45 = 9$$

$$45 + 9 = 54$$

To decrease 220 by 5%

$$5\% \text{ of } 220 = 11$$

$$220 - 11 = 209$$

### Using a multiplier

#### Examples

To increase 45 by 20%

After the increase you will have  $100\% + 20\% = 120\% = 1.2$

$$1.2 \times 45 = 54$$

To decrease 220 by 5%

After the decrease you will have  $100\% - 5\% = 95\%$

$$0.95 \times 220 = 209$$

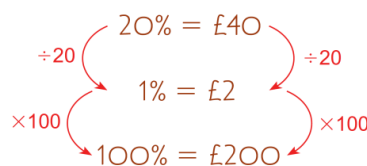
### Finding the original amount

#### Using arrow diagrams

#### Worked example

20% of an amount is £40.

Work out the original amount.



Source: KS3 Maths Progress

### Using function machines

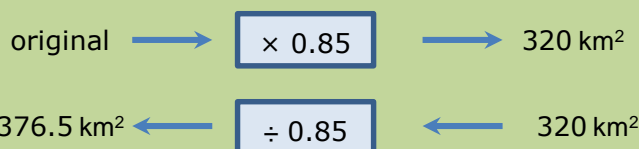
#### Example

When studying ecosystems, for example, consider the area of a woodland that has been reduced by 15% because of deforestation.

The final area is 320 km<sup>2</sup>.

Calculate the original area.

To calculate the area after 15% decrease, you would multiply by 0.85:



### Percentage change

$$\text{percentage change} = \frac{\text{actual change}}{\text{original amount}} \times 100$$

#### Example

When studying development, for example, in 2010 a region's annual GDP was £80m.

In 2014, the same region has increased its economic output to £120m.

The actual increase in GDP is  $120 - 80 = £40\text{m}$ .

The fractional increase is  $\frac{\text{actual increase}}{\text{original price}} = \frac{40}{80}$

$\frac{40}{80}$  as decimal is 0.5

Percentage increase is  $0.5 \times 100 = 50\%$

## Terminology

- Percent means 'out of 100'. A percentage is a fraction with a denominator of 100.
- You can calculate percentages of amounts, e.g. 20% of \$500.
- You can write one number as a percentage of another, e.g. write  $\frac{7}{50}$  as a percentage.

## 3.3 Converting between fractions, decimals and percentages

### Demand

All KS3 students will learn how to convert between fractions, decimals and percentages.

### Approach

#### Percentage to fraction to decimal

$$40\% = \frac{40}{100} = 0.4$$

#### Decimal to percentage

Multiply by 100:

$$0.3 = 30\%$$

$$0.02 = 2\%$$

#### Percentage to decimal

Divide by 100:

$$62\% = 0.62$$

$$7.5\% = 0.075$$

#### Simple fractions to percentages

Multiply or divide both numbers to get a fraction with a denominator of 100:

$$\begin{array}{c} \times 5 \\ \frac{3}{20} = \frac{15}{100} = 15\% \\ \times 5 \end{array}$$

$$\begin{array}{c} \div 2 \\ \frac{62}{200} = \frac{31}{100} = 31\% \\ \div 2 \end{array}$$



## Convert a fraction to a percentage

Convert the fraction to a decimal, then convert the decimal to a percentage.

For example:

$$\frac{34}{80} = 0.425 = 42.5\%$$

Students can input  $\frac{34}{80}$  as a fraction into a scientific calculator and press = (or the S-D button on some calculators) to get the equivalent decimal.

## Terminology

- When converting decimals to percentages or vice versa, do not say 'move the decimal point two places'. Instead, say 'multiply by 100' or 'divide by 100' as appropriate. For example:

$$0.52 = 52\%$$

$$3\% = 0.03$$

## 3.4 Ratios

### Demand

Students learn to simplify ratios, and write them in the form  $1 : n$  or  $n : 1$  in KS3.

Students learn to relate ratios to fractions in KS3, but many continue to make errors with this type of calculation.

### Approach

#### Simplifying ratios

A ratio in its simplest form only contains whole number values.

Divide all the numbers in the ratio by the highest common factor:

$$\begin{array}{ccc} & 2 : 6 & \\ \div 2 \swarrow & & \searrow \div 2 \\ & 1 : 3 & \end{array}$$

$$\begin{array}{ccc} & 6 : 15 & \\ \div 3 \swarrow & & \searrow \div 3 \\ & 2 : 5 & \end{array}$$

This ratio is not in its simplest form, because the two numbers both still have a common factor, 2:

$$\begin{array}{ccc} & 8 : 20 & \\ \div 2 \swarrow & & \searrow \div 2 \\ & 4 : 10 & \end{array}$$

**Writing in the form 1 : n (sometimes called a unit ratio)**

Divide both numbers by the first number in the ratio:

$$\begin{array}{c} 5 : 7 \\ \div 5 \quad \quad \div 5 \\ \hline 1 : 1.4 \end{array}$$

**Writing in the form n : 1**

Divide both numbers by the second number in the ratio:

$$\begin{array}{c} 20 : 12 \\ \div 12 \quad \quad \div 12 \\ \hline 1.67 : 1 \end{array}$$

**Comparing ratios**

Write both ratios in the form 1 : n or n : 1.

**Example**

When studying development. For example, in Country A there are 20 people who are unemployed for every 120 people who are economically active.

In Country B there are 15 people who are unemployed for every 85 people who are economically active.

Which country has more unemployment?

Country A

$$\begin{array}{c} 20 : 120 \\ \div 20 \quad \quad \div 20 \\ \hline 1 : 6 \end{array}$$

Country B

$$\begin{array}{c} 15 : 85 \\ \div 15 \quad \quad \div 15 \\ \hline 1 : 5.7 \end{array}$$

Country B has a lower ratio, so as a proportion there are more unemployed people.

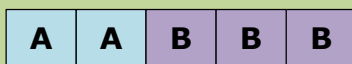
**Ratio and proportion**

When studying coastal landscapes. For example, a cliff is composed of two different rock types (geologies), A and B, in the ratio 2 : 3. What fraction of the cliff is:

a) Geology A?

b) Geology B?

Draw a bar model to illustrate the mixture:



$\frac{2}{5}$  is A and  $\frac{3}{5}$  is B

### Terminology

Write the ratio of  $A$  to  $B$  means write  $A : B$ . If you want students to write the ratio as  $\frac{A}{B}$  you need to say 'write the ratio as  $\frac{A}{B}$ '.

To simplify a ratio, divide all the numbers in the ratio by their highest common factor.

To compare ratios, write them in the form  $1 : n$ , or  $n : 1$ . This is sometimes called a unit ratio.

A **ratio** compares two quantities, and translates into a statement such as 'for every 3 black there are 2 red'.

A **proportion** compares a part with a whole. A proportion can be given as a fraction or a percentage.

#### Common error

Students look at  $2 : 3$  and think the fraction is  $\frac{2}{3}$ .

## 4. Use angular measures in degrees

---

### Demand

All KS3 students learn to measure angles in degrees, and to calculate missing angles in simple diagrams.

### Terminology

- Angles around a point add up to  $360^\circ$ , angles on a straight line add up to  $180^\circ$ .
- A right angle is  $90^\circ$ .
- Perpendicular lines meet at  $90^\circ$ .
- Students can use a circular protractor to measure angles greater than  $180^\circ$ .
- Angles less than  $90^\circ$  are acute; angles between  $90^\circ$  and  $180^\circ$  are obtuse; angles greater than  $180^\circ$  are reflex angles.

## 5. Visualise and represent 2-D and 3-D forms, including two-dimensional representations of 3-D objects

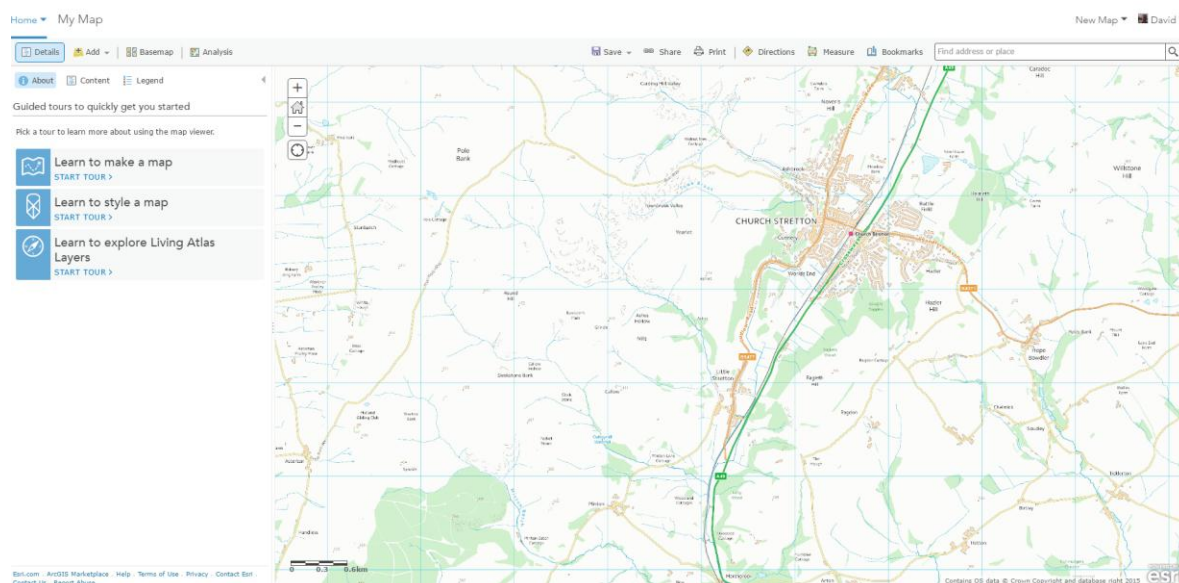
### Demand

All KS3 students learn about simple nets.

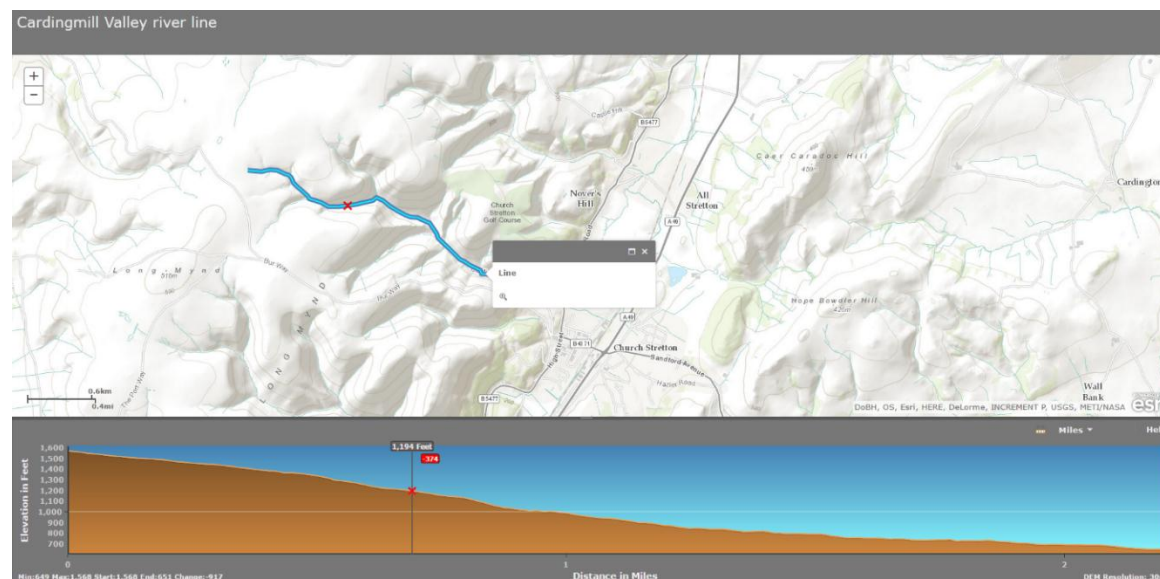
### Terminology

- 2-D representations of 3-D shapes include 3-D sketches, accurate 3-D drawings on isometric paper, nets, and plans and elevations.
- A net is the 2-D shape that can be folded up to make a 3-D shape.

The plan view is the view from above an object (like an OS map). The side elevation is the view from one side and the front elevation is the view from the front. GIS could then be used to create a cross section.



**Figure 24:** Plan view of an area. Source: My Map (Google)



**Figure 25** Using GIS to show the long profile of a river. Source: ESRI

## 6. Calculate areas of triangles and rectangles, surface areas and volumes of cubes

### Demand

All KS3 students learn to calculate the area of a rectangle and triangle.

KS3 top and middle sets maths students use hectares.

All KS3 students calculate surface area and volume of cubes and cuboids.

### Approach

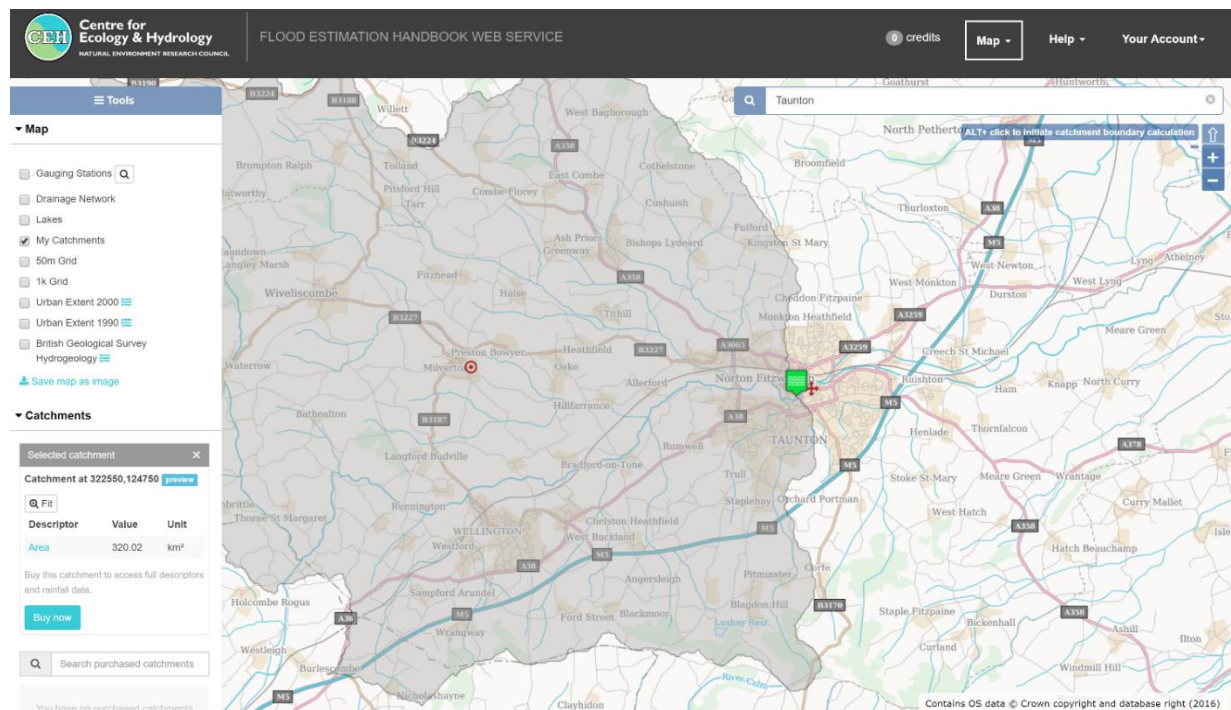
#### Estimating the area of an irregular shape – counting squares

To calculate the area of an irregular shape, such as a river catchment area, draw round the shape on graph paper that has small squares. Then count the squares inside the area. For squares that cross the perimeter, count those that are more than half-in as whole ones, and don't count those that are more than half-out. This will give you an **estimate** of the area. The smaller the graph squares, the more accurate the estimate.

Remember that area is measured in square units, such as  $\text{km}^2$ ,  $\text{m}^2$  or  $\text{cm}^2$ .

You can use an online free GIS resource to verify and check you answer, such as the Flood Estimation Handbook Service. A free account is required to register:

<https://fehweb.ceh.ac.uk/GB/map>. Of course, other GIS programs could also do this, e.g. ArcGIS Online



**Figure 23** An example of a river catchment area. Note the area is shown in the bottom left of the web-page – 320km² Source: Centre for Ecology and Hydrology

## Calculate the area of a rectangle

Area of a rectangle = length  $\times$  width

$$A = l \times w \text{ or } A = lw$$

## Calculate the area of a triangle

This formula works for all triangles, not just right-angled ones.

$h$  is the perpendicular height.

Area of triangle =  $\frac{1}{2} \times \text{base} \times \text{height}$

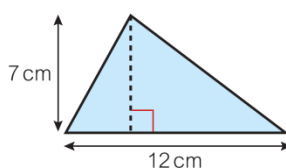
$$A = \frac{1}{2}bh$$

### Worked example

Work out the area of this triangle.

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2} \times 12 \times 7 \\ &= 42 \text{ cm}^2 \end{aligned}$$

Write the formula, then substitute the numbers into the formula.



### Key point

Area of a triangle =  $\frac{1}{2} \times \text{base length} \times \text{perpendicular height}$   
which can be written as  $A = \frac{1}{2}bh$ .  
The height measurement must be perpendicular (at  $90^\circ$ ) to the base.

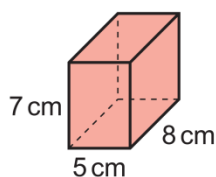
Source: KS3 Maths Progress

In a right-angled triangle, the two sides that meet at the right angle are the base and the height.

## Calculate surface area by drawing the net

### Example 4

Work out the surface area of this cuboid.



Sketch the net.

Label the lengths.

Work out the area of each face.

	8 cm	5 cm	
7 cm	35 cm <sup>2</sup>	8 × 5 = 40 cm <sup>2</sup>	5 cm
	5 cm	8 × 7 = 56 cm <sup>2</sup>	7 cm
		7 × 5 = 35 cm <sup>2</sup>	
		40 cm <sup>2</sup>	5 cm
		56 cm <sup>2</sup>	7 cm
	8 cm		

Total surface area = 40 + 56 + 40 + 56 + 35 + 35 = 262 cm<sup>2</sup>

Source: Edexcel GCSE (9-1) Mathematics

## Terminology

- Use the mathematical terms rectangle and rectangular (instead of oblong) and rhombus (instead of diamond).
- Use the mathematical term cuboid (instead of box-like).
- Specify the shape of an object when you ask students to calculate the area. If it is a rectangle, say so clearly. Otherwise this reinforces a common misconception that the area of any shape is length  $\times$  width. For example, 'Estimate the abundance of foxgloves in an area 60 m long and 10 m wide' is not accurate enough. Tell them it is a rectangular area.
- The perimeter of a 2-D shape is the distance all around the outside.

• The area of a 2-D shape is the amount of space inside the shape. It is measured in squared units,  $\text{mm}^2$ ,  $\text{cm}^2$ ,  $\text{m}^2$ , hectares (1 ha = 10 000  $\text{m}^2$ ) and  $\text{km}^2$ . Shifting between scales and areas is important in geography.

- You can estimate the area of an irregular shape by drawing around it on cm squared paper and counting the squares.
- If a shape is close to a rectangle, you can estimate the area by approximating it to a rectangle.
- The surface area of a 3-D shape is the total area of all the surfaces added together.
- In maths we use 'area' for 2-D shapes (e.g. a rectangular section of beach) and 'surface area' for 3-D shapes, because it is the area of all the surfaces added together.
- To calculate the surface area of a cuboid, find the areas of all the faces and add them together.

## Inconsistency

In maths we do not give students a formula for the surface area of a cuboid.

- The volume of a 3-D shape is the amount of space it takes up. It is measured in cubed units,  $\text{mm}^3$ ,  $\text{cm}^3$ ,  $\text{m}^3$ .
- Capacity is the amount of liquid a 3-D solid can hold. It is measured in ml or litres.



## 7. Units and compound units, including conversion between units

### Demand

All students meet the prefixes for metric units in GCSE maths. The only ones they are likely to use frequently in maths are kg, km, cm, ml, mm.

All students learn to convert between metric units of area and volume in GCSE maths.

All students learn to convert speeds in m/s to km/h and vice versa in GCSE maths.

Students are not expected to know metric/imperial conversions, or imperial-to-imperial conversions such as lbs to stones, or feet to yards.

### Approach

#### Use arrow diagrams and function machines for simple conversions

'By counting the number in 15 seconds and multiplying by 4'

Rather than present this as a 'magic' formula, it would be good to get students to work out how many 15 seconds there are in a minute, and so what to multiply by.

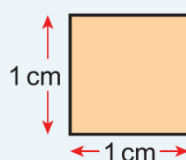
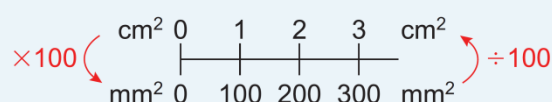
#### Area conversions

Students may not remember area conversion factors, but will learn in GCSE maths how to work them out, as follows.

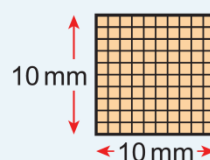
You can use a double number line to convert between area measures.

#### Key point 4

These two squares have the same area.  
To convert from  $\text{cm}^2$  to  $\text{mm}^2$ , multiply by 100.  
To convert from  $\text{mm}^2$  to  $\text{cm}^2$ , divide by 100.



$$\text{Area} = 1 \text{ cm} \times 1 \text{ cm} = 1 \text{ cm}^2$$

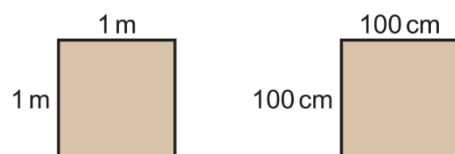


$$\text{Area} = 10 \text{ mm} \times 10 \text{ mm} = 100 \text{ mm}^2$$

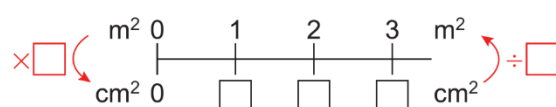
Source: Edexcel GCSE (9-1) Mathematics

This question shows students how to convert  $\text{km}^2$  to  $\text{m}^2$  and vice versa.

Use these diagrams to help you work out the number of  $\text{cm}^2$  in  $1 \text{ m}^2$ .



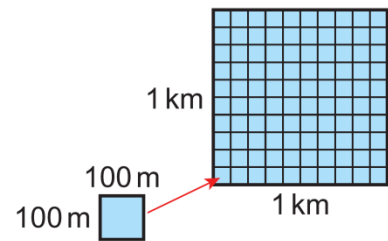
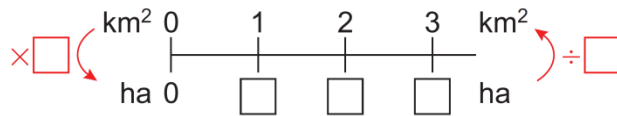
Copy and complete the double number line.



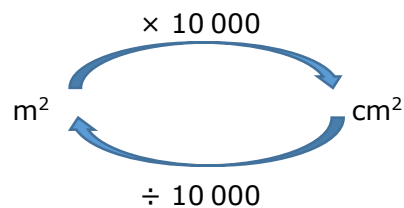
This question shows hectares to  $\text{km}^2$  and vice versa. This is important in the use of OS maps in geography.

The diagram shows  $1 \text{ km}^2$  divided into  $100 \text{ m}$  squares.

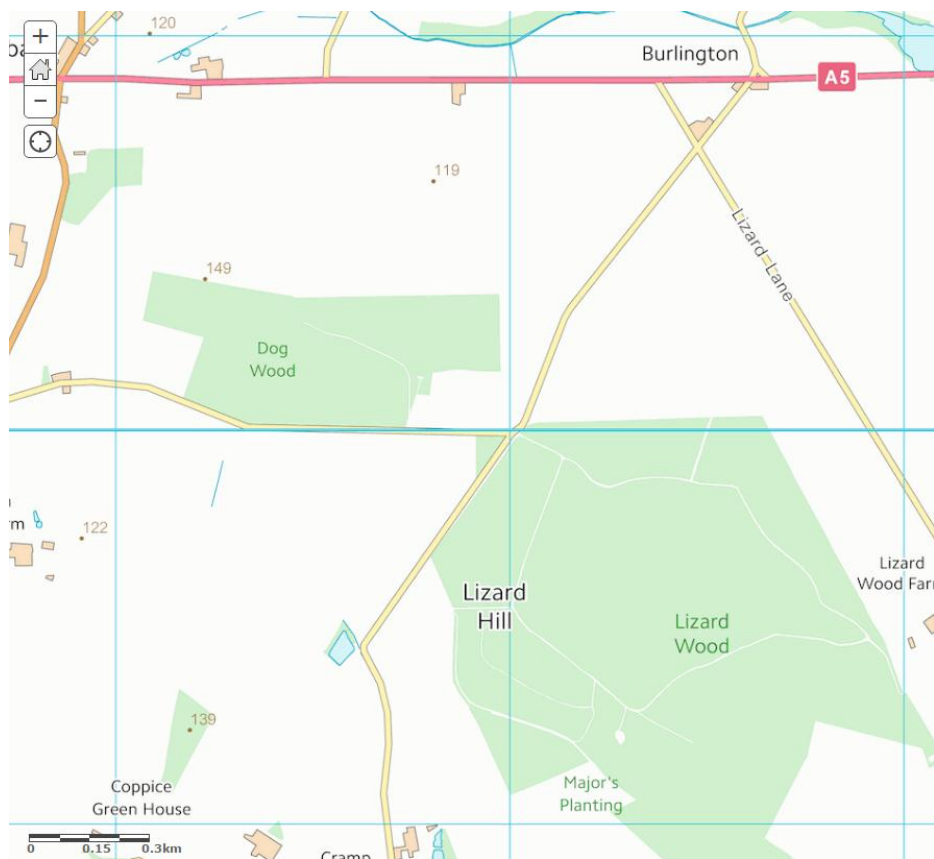
- a What is the area of each  $100 \text{ m}$  square?
- b How many hectares are there in  $1 \text{ km}^2$ ?
- c Copy and complete the double number line.



You can use arrow diagrams to help convert between area measures.



In this geography example, the blue grid lines on the maps are  $1 \text{ km}$  apart. Lizard wood (in the extract) is approximately  $1 \text{ km}^2$ , or  $100 \text{ ha}$ . Dog Wood takes up about 20% of a single square, so would be  $20 \text{ ha}$  or  $200\,000 \text{ m}^2$ .



**Figure 26** A map showing areas of woodland. Source: Ordnance Survey

## Terminology

**Inconsistency:** in maths, science as well as geography we use cm, mm, kg, etc. These are abbreviations, not symbols.

- We use the abbreviations min (not m) for minutes.
- For compound units we use m/s, km/h, g/cm<sup>3</sup>, rather than ms<sup>-1</sup> etc.
- In maths books, where we use 'per' we give a literacy hint, e.g. 8 g/cm<sup>2</sup> means 8 grams in every cm<sup>2</sup>.
- Area is measured in squared units: mm<sup>2</sup>, cm<sup>2</sup>, m<sup>2</sup>, hectares (1 ha = 10 000 m<sup>2</sup>) and km<sup>2</sup>.
- Volume is measured in cubed units: mm<sup>3</sup>, cm<sup>3</sup>, m<sup>3</sup>.
- Capacity is measured in litres and ml. Students do not use dl in maths. Some students may use cl.

- 1 cm<sup>3</sup> = 1 ml. This might be relevant in geography when dealing with fieldwork data on rivers, for example. Conventionally, discharge is described as m<sup>3</sup>/sec (m<sup>3</sup>/s<sup>-1</sup>), but for smaller rivers it may better to express it as litres/second, or l/sec.

- Some students meet the prefixes for metric units in KS3 maths, e.g. M stands for Mega and means 10<sup>6</sup>.
- It is easier to convert measures to the units required before doing a calculation, than to convert the answer into the units required.
- If students are required to convert between metric and imperial units, they should be given the conversion factor. In GCSE maths they are not expected to know metric/imperial equivalents.

## 8. Significant figures, decimal places, accuracy

### Demand

All KS3 students will have learned to round to the nearest whole number and 1, 2 or 3 dp. They should be able to cope with rounding to more dp as an extension of rounding to 3 dp.

### Significant figures

At GCSE only Higher tier students learn about upper and lower bounds, and percentage error.

For percentage error they answer questions such as: Given a percentage error of  $\pm 10\%$ , what is the largest/smallest possible value?

Answering questions such as 'What is the percentage error?' for a given value is not covered in the GCSE maths specification.

### Approach

Look at the digit *after the* last one you want to keep. Round up if this digit is 5 or more; round down if it is 4 or less.

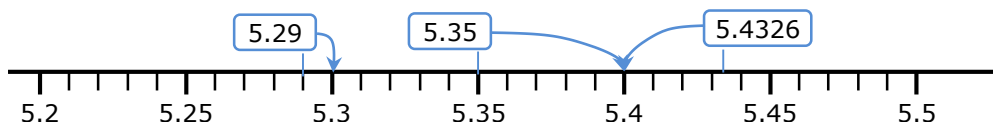
### Rounding to 1 dp

5.4|326  
↓  
5.4 (1 d.p.)  
less than 5  
round down

5.2|91  
↓  
5.3 (1 d.p.)  
5 or more  
round up

5.3|5  
↓  
5.4 (1 d.p.)  
5 or more  
round up

On a number line, round to the nearest value with 1 decimal place:



35.42|9  
↓  
35.43 (2 d.p.)  
5 or more  
round up

126.37|2  
↓  
126.37 (2 d.p.)  
less than 5  
round down

### Rounding to 3 dp

0.053|21  
↓  
0.053 (3 d.p.)  
less than 5  
round down

11.291|5  
↓  
11.292 (3 d.p.)  
5 or more  
round up

## Rounding to significant figures

### Small numbers

1st significant figure  
= 4 ten thousandths  
↓  
0.000 483

### Large numbers

1st significant figure  
= 5 billion  
↓  
518 376 000

### Round to 2 significant figures (2 s.f.)

0.00048|3  
    ↓  
    less than 5  
    round down  
  
0.00048

51|8 376 000  
    ↓  
    5 or more  
    round up  
  
520 000 000  
    ↓  
    Add zeroes so the 5 is still  
    in the 'billions' position

## Upper and lower bounds calculations

Find the upper and lower bounds of the given values, before doing the calculation.

### Terminology

- The number of **decimal places** is the number of digits after the decimal point. So, 10.5219 has 4 decimal places, and 10 has no decimal places.
- In any number the first **significant figure** is the one with the highest place value. It is the first non-zero digit counting from the left. **Inconsistency:** Zero is counted as a significant figure if it is between two other non-zero significant figures. Other zeros are place holders – if you took them out the place value of the other digits would change.

place holders  
↓ ↓  
0 . 0 0 5 0 7 3  
    ↑ ↑ ↑ ↑  
    1st 2nd 3rd 4th  
significant figures

place holders  
↓ ↓ ↓  
3 9 0 4 1 2 0 0 0  
↑ ↑ ↑ ↑ ↑ ↑  
1st 2nd 3rd 4th 5th 6th  
significant figures

- To round a number to a given number of significant figures or decimal places, look at the digit after the last one you need. Round up if the digit is 5 or more, and round down if the digit is 4 or less.

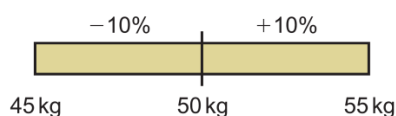
- **Inconsistency:** Rounding numbers reduces accuracy. Your results cannot be more accurate than your starting values. In geography, in calculations your answer cannot have more significant values than the numbers in the calculation. In maths we tell students not to give more decimal places in the answer than in the calculation, and also to consider if their answers are practical – e.g. could you measure 4.321 cm to that level of accuracy? In geography, where fieldwork equipment *may be* more accurate, the answer to this may be 'yes'.

- 8.95 rounded to 1 decimal place is 9.0. You must write the '.0' to show the value in the decimal place.
- When a value is rounded, its true value lies within half a unit either side of the rounded value.  
The values of  $x$  that round to 5.2 to 1 dp are  
 $5.15 \leq x < 5.25$
- Note, this means that the highest value that rounds to 5.2 to 1 dp is 5.249999....
- The upper bound is half a unit greater than the rounded measurement.  
The lower bound is half a unit less than the rounded measurement.

$$\begin{array}{cc} 12.5 \leq x < 13.5 \\ \text{lower} & \text{upper} \\ \text{bound} & \text{bound} \end{array}$$

NB Only Higher tier maths students learn this.

- To determine an appropriate level of accuracy for an answer to a calculation, you can find the upper and lower bounds of the calculation.  
E.g. if upper bound is 28.42896  
and lower bound is 28.42712  
then 28.42 is a suitable level of accuracy.
- NB: Only Higher tier maths students learn this.
- A 10% error interval means that a value could be up to 10% larger or smaller than the value given.



- You can write an error interval as an inequality:  $45 \leq m \leq 55$  kg.

## 9. Standard form

### Demand

All KS3 students learn to write numbers in index form and use the index laws for multiplication and division.

All GCSE students learn the positive and negative powers of 10. Foundation students often find the negative and zero powers difficult to understand/ remember, as they are the only negative and zero powers they use. Higher students use negative and zero indices with a range of numbers so are likely to have a better understanding.

All GCSE students learn to read and write very small and very large numbers in standard form.

### Approach

#### Calculating powers of 10

Follow a pattern:

$$\begin{aligned}
 10^1 &= 10 \\
 10^2 &= 10 \times 10 = \square \\
 10^3 &= 10 \times 10 \times 10 = \square \\
 10^4 &= 10 \times 10 \times 10 \times 10 = 10\,000 \\
 10^5 &= \dots \\
 10^6 &= \dots \\
 10^3 &= 1000 \\
 10^2 &= 100 \\
 10^1 &= 10 \\
 10^0 &= 1 \\
 10^{-1} &= \frac{1}{10} = 0.1 \\
 10^{\square} &= \frac{1}{100} = \frac{1}{10^2} = 0.01 \\
 10^{\square} &= \square = \square = \square
 \end{aligned}$$

#### Writing large numbers in standard form

These examples are taken from *Edexcel GCSE (9-1) Mathematics Foundation* student book.

##### Example 4

Write 4000 in standard form.

$$\begin{aligned}
 4000 &= 4 \times 1000 \\
 &= 4 \times 10^3
 \end{aligned}$$

Write the number as a number between 1 and 10 multiplied by a power of 10.

Write the power of 10 using indices.

##### Example 5

Write 45 600 in standard form.

$$45\,600 = 4.56 \times 10^4$$

4.56 lies between 1 and 10.  
Multiply by the power of 10 needed to give the original number.

4 5 6 0 0

## Writing small numbers in standard form

### Example 6

Write 0.00005 in standard form.

$$0.00005 = 5 \times 0.00001$$

$$= 5 \times 10^{-5}$$

Write the number as a number between 1 and 9 multiplied by a power of 10.

### Key point 7

To write a small number in standard form:

- Place the decimal point after the first non-zero digit.
- How many places has this moved the digit? This is the negative power of 10.

### Example 7

Write 0.00352 in standard form.

$$0.00352 = 3.52 \times 10^{-3}$$

3.52 lies between 1 and 10.  
Multiply by the power of 10 needed to give the original number.

0.00352

## Calculating with numbers in standard form

### Multiplication and division

#### Example 3

Work out  $(5 \times 10^3) \times (7 \times 10^6)$

$$5 \times 7 \times 10^3 \times 10^6$$

Rewrite the multiplication grouping the numbers and the powers.

$$35 \times 10^9$$

Simplify using multiplication and the index law  $x^m \times x^n = x^{m+n}$ .  
This is not in standard form because 35 is not between 1 and 10.

$$35 = 3.5 \times 10^1$$

Write 35 in standard form.

$$35 \times 10^9 = 3.5 \times 10^1 \times 10^9 = 3.5 \times 10^{10}$$

Work out the final answer.

Work out  $\frac{2.4 \times 10^5}{3 \times 10^2}$

$$= 0.8 \times 10^3$$

$$= 8 \times 10^2$$

Divide 2.4 by 3.  
Use the index law  $x^m \div x^n = x^{m-n}$  to divide  $10^5$  by  $10^2$ .

### Addition and subtraction

Write numbers in decimal form before adding and subtracting.

Write the answer in standard form.

Work out  $3.6 \times 10^2 + 4.1 \times 10^{-2}$

$$= 360 + 0.041$$

$$= 360.041$$

$$= 3.60041 \times 10^2$$



Work out  $2.5 \times 10^6 - 4 \times 10^4$

$$\begin{array}{r} 2\,500\,000 \\ - \quad 40\,000 \\ \hline 2\,460\,000 \\ 2.46 \times 10^6 \end{array}$$

## Terminology

- Any number can be raised to a power or index. The power or index tells you how many times the number is multiplied by itself.  $3^4 = 3 \times 3 \times 3 \times 3$
- We read  $3^4$  as '3 to the power 4'.
- Some calculators have a power or index key. In maths we do not tell students which key presses to use, as calculators vary. Instead we would say 'Make sure you know how to input numbers raised to a power on your calculator.'
- Any number raised to the power zero = 1.
- The index laws:
  - to multiply powers of the same number, add the indices.
  - to divide powers of the same number, subtract the indices.
- Some of the powers of 10:

$10^{-4}$	$10^{-3}$	$10^{-2}$	$10^{-1}$	$10^0$	10	$10^2$	$10^3$	$10^4$
0.0001 or $\frac{1}{10\,000}$	0.001 or $\frac{1}{1000}$	0.01 or $\frac{1}{100}$	0.1 or $\frac{1}{10}$	1	10	100	1000	10 000

- Standard form is a way of writing very large or very small numbers as a number between 1 and 10 multiplied by a power of 10.  
 $A \times 10^n$  where  $A$  is between 1 and 10 and  $n$  is the power of 10

• **Inconsistency:** When writing numbers in standard form, do not talk about 'moving the decimal point'. The position of the decimal point remains fixed. Multiplying by a power of 10 moves digits places to the left and dividing by a power of 10 moves digits places to the right.

- On some calculators you can enter numbers in standard form, or answers may be given in standard form. In maths we do not tell students which key presses to use, as calculators vary. Instead we would say 'Make sure you know how to enter and read numbers in standard form on your calculator.'

## 10. Statistical skills – average, mean, quartiles/percentiles

### Demand

Normal distribution is not covered in maths GCSE.

Foundation level students do not use quartiles and interquartile range.

Higher level students learn quartiles and interquartile range in year 11.

Percentiles are not on the GCSE maths specification.

Only Higher level students find means and medians from bar charts and histograms in GCSE maths.

### Approach

#### Calculating the range

##### *From a small data set*

Range = largest value minus smallest value

For example, for this data: 2, 2, 5, 7, 2, 4, 6, 9

the range is  $9 - 2 = 7$

##### *From a frequency table*

Range = largest value minus smallest value

Number of cars per household	Frequency
1	2
2	15
3	6

Range =  $3 - 1 = 2$

NB: It is the range of the data values, not of the frequencies.

##### *From a grouped frequency table*

An estimate of the range is the largest possible value minus smallest possible value

#### Worked example

In a survey, people were asked their age. The table shows the results.

Age, $a$ (years)	Frequency
$0 \leq a < 10$	12
$10 \leq a < 20$	15
$20 \leq a < 30$	2
$30 \leq a < 40$	11

Work out an estimate for the range of ages.

From the frequency table, the smallest possible age is 0 years.

The largest possible age is 40 years.

So an estimate of the range is  $40 - 0 = 40$  years.

Source: *KS3 Maths Progress*

## Finding the mode

### From a data set

For the data set 2, 2, 5, 7, 2, 4, 6, 9, the mode is 2.

For the data set 1, 1, 3, 4, 2, 5, 3, 3, 2, 2, 1 the modes are 1 and 3.

### From a frequency table

For this data, the mode is 2 cars per household.

Number of cars per household	Frequency
1	2
2	15
3	6

#### Common error

Students may give '15' as the mode (the highest frequency), rather than 2, which is the number of household cars with the greatest frequency.

### From a grouped frequency table

For this data the modal class is  $10 \leq a < 20$

Age, $a$ (years)	Frequency
$0 \leq a < 10$	12
$10 \leq a < 20$	15
$20 \leq a < 30$	2
$30 \leq a < 40$	11

## Finding the median and quartiles

First write the data in order.

0    2    4    5    7    8    10    12    12    12    13    14    14    15    16

↑  
The lowest quartile  
is the 4th value

↑  
The median is the  
8th value











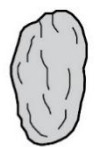

↑  
The upper quartile  
is the 12th value

#### Common error

Students may not order the data before finding the median and quartiles.

The information below has been taken from a frequency table of fieldwork data from the coast (based on length of long axis).

Classification of sediment shape	Frequency
1 (Very rounded)	4
2	8
3	6
4	2
Total	20

High Sphericity						
Low Sphericity						
	Very Angular	Angular	Subangular	Poorly Rounded	Rounded	Very Rounded

There are 20 pieces of data in the table.

The median is the  $\frac{20+1}{2} = 10.5$ th data item (i.e. between the 10th and 11th items).

Classification of sediment shape	Frequency
1	4
2	8
3	6
4	2
Total	20

4

$4 + 8 = 12$

Add up the frequencies to find the 10th and 11th data items.

The 10th and 11th items are both shape 2 examples, so the median is shape classification 2.

### Calculating the quartiles from a frequency table

The lower quartile is the  $\frac{20+1}{4} = 5.25$ th data item (i.e. between the 5th and 6th items).

The upper quartile is the  $\frac{3(20+1)}{4} = 15.75$ th data item (i.e. between the 15th and 16th items).

Classification of sediment shape	Frequency
1	4
2	8
3	6
4	2
Total	20

4

$4 + 8 = 12$

$12 + 6 = 18$

Add up the frequencies to find the 5th and 6th data items and the 15th and 16th data items.

The 5th and 6th items are both classification 2, so the lower quartile is 2.

The 15th and 16th items are both classification 3, so the upper quartile is 3.

### Finding the interval containing the median from a grouped frequency table

Age, $a$ (years)	Frequency
$0 \leq a < 10$	12
$10 \leq a < 20$	15
$20 \leq a < 30$	2
$30 \leq a < 40$	11

$$12$$

$$12 + 15 = 27$$

Add up the frequencies to find the 20th and 21st data items.

Total frequency = 40

$$\text{Median} = \frac{40+1}{2} = 20.5\text{th data item.}$$

The 20th and 21st data items are in the interval  $10 \leq a < 20$ .

### Calculating the mean of a small data set

#### Example 1

Work out the mean of 3, 6, 7, 7 and 8.

$$3 + 6 + 7 + 7 + 8 = 31$$

Add the values first to find the total.

$$\frac{31}{5} = 6.2$$

The mean is 6.2

There are 5 values, so divide the total by 5.

Source: Edexcel Mathematics (9-1) Foundation

### Measures of central tendency

When using a calculator to calculate a mean, students may add the numbers and not press = before dividing, which will give an incorrect value.

### Calculating the mean from a frequency table

#### Worked example

Jack asked students in his class how many pets they had. Here are his results. Work out the mean.

Number of pets	Frequency	Total number of pets
0	7	$0 \times 7 = 0$
1	8	$1 \times 8 = 8$
2	6	$2 \times 6 = 12$
3	3	$3 \times 3 = 9$
4	1	$4 \times 1 = 4$
Total	25	33

Add a column to the table to work out the total numbers of pets.

Work out the total frequency (number of people in the survey) and the total number of pets.

$$\text{mean} = 33 \div 25 = 1.32$$

$$\text{mean} = \text{total number of pets} \div \text{number of people}$$

Source: KS3 Maths Progress

## Calculating an estimate of the mean from a grouped frequency table

### Worked example

In a survey, people were asked their age. The table shows the results.

Age, $a$ (years)	Frequency
$0 \leq a < 10$	12
$10 \leq a < 20$	15
$20 \leq a < 30$	2
$30 \leq a < 40$	11

Calculate an estimate for the mean age.

Age, $a$ (years)	Frequency	Midpoint of class	Midpoint $\times$ Frequency
$0 \leq a < 10$	12	$\frac{0 + 10}{2} = 5$	$5 \times 12 = 60$
$10 \leq a < 20$	15	$\frac{10 + 20}{2} = 15$	$15 \times 15 = 225$
$20 \leq a < 30$	2	25	$25 \times 2 = 50$
$30 \leq a < 40$	11	35	$35 \times 11 = 385$
<b>Total</b>	<b>40</b>		<b>720</b>

Add a column to calculate an estimate of the total age for each class.

Calculate the total number of people in the survey and the sum of their ages.

mean = sum of ages  $\div$  total number of people

$$= \frac{720}{40}$$

$$= 18$$

Source: KS3 Maths Progress

## Calculating means and medians from bar charts and histograms

Make a frequency table for the bar chart or histogram, and use the appropriate method shown above.

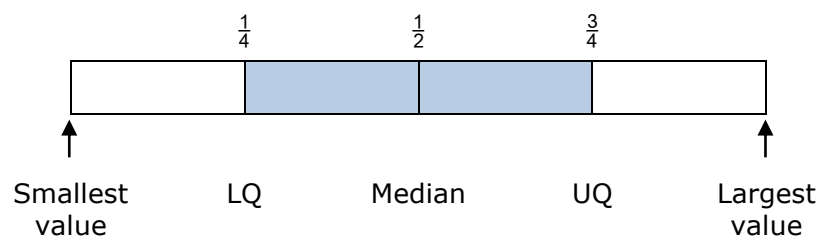
### Terminology

- Mean, median and mode are all averages. In everyday life when someone says 'average' they are usually talking about the mean. In geography, students need to be aware of the importance of this term to describe data, as well as its advantages and disadvantages.
- The range is a measure of spread. It is calculated as largest value – smallest value. Note that the range is a single number, for example, not 3-12, two numbers separated by a hyphen. In a maths question we would say 'Work out' the range – you need to do a calculation to find it.
- A larger range means the data is less consistent. A smaller range means the data is more consistent.
- You can estimate the range from a grouped frequency table, as largest possible value minus smallest possible value.
- Mean =  $\frac{\text{total of all the values}}{\text{number of values}}$
- You can calculate the mean from an ungrouped frequency table. For a grouped frequency table, you can calculate an estimate of the mean (because you use the midpoint of each group as an estimate of the data values in that group). Please word such questions as 'Calculate an estimate of the mean'.

- The mode is the most common value. In a frequency table, this is the value with the highest frequency. The mode is one of the data values. A set of data can have more than one mode. For grouped data, the modal class is the class interval with the highest frequency.
- The median is the middle value when the data is written in order. It may not be one of the data values (e.g. it could be halfway between two values).
- For an ordered set of data with an even number of values, the median is the mean of the two middle values (which is the same as the value midway between them).
- For a set of  $n$  items of data, the median is the  $\frac{n+1}{2}$ th data item. When  $n$  is very large, you can use the  $\frac{n}{2}$ th data item.

- If you have an anomalous value (sometimes called an outlier in maths), i.e. one that is likely to have been a recording error, you can ignore this when calculating the mean. In geography students often think anomalies are 'incorrect', especially when linked to fieldwork. This may not be the case, but instead the anomalies may be valid, albeit unexpected. values that require consideration.

- For a set of ordered data, the median is the value halfway through the data. The lower quartile is the value one quarter of the way into the data set. The upper quartile is the value three quarters of the way into the data set.
- For a set of  $n$  items of data, the lower quartile is the  $\frac{n+1}{4}$ th data item and the upper quartile is the  $\frac{3(n+1)}{4}$ th data item. When  $n$  is very large, you can use the  $\frac{3n}{4}$ th data item.
- The interquartile range is value calculated by: upper quartile minus lower quartile. It shows how spread out the middle 50% of the data is.



- The 50th percentile is the median. The 10th percentile is the value 10% of the way into the data set, when the data is in order.
- NB: Students do not learn about percentiles in GCSE maths.
- To describe a set of data you should give at least one average and a measure of spread.
- To compare two sets of data you should compare one average and one measure of spread.

## 11. Sampling and data

### Demand

Students learn that interpolation is more accurate than extrapolation, though may not use these terms to describe it.

Students learn to write and criticise questionnaire questions in KS3, but this is not on the GCSE specification.

All students learn about choosing a random sample to avoid bias in GCSE maths. They should understand the concept of random numbers and using these to generate a random sample, but they may not know how to generate random numbers on a calculator.

Only Higher level students learn stratified sampling and the capture-recapture method in GCSE maths.

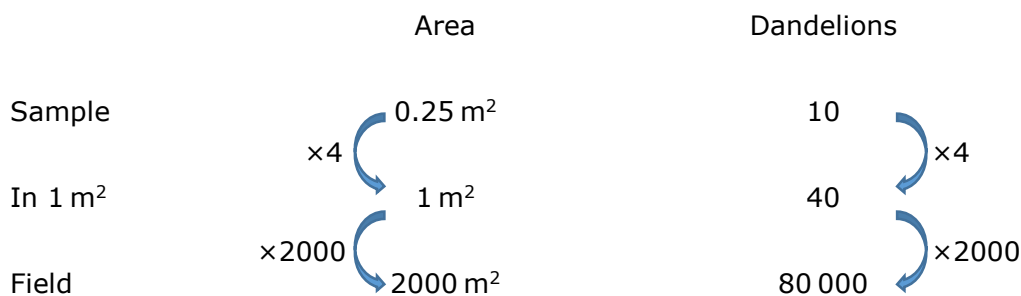
### Approach

#### Estimating from samples

##### *Use arrow diagrams to scale up*

For example a student is planning some fieldwork to investigate differences in dandelion plants between two fields. The dandelions are being used as one indicator of tourism pressure throughout both areas, so this sampling method is being used to estimate how many dandelions there might be in total in each of the fields. E.g. in the first field a quadrat  $0.5\text{ m} \times 0.5\text{ m}$  is randomly thrown 5 times. The number of dandelions in each quadrat is counted. The mean is 10 plants. The field has area  $2000\text{ m}^2$ .

Estimate the number of dandelions in the field.



### Terminology

#### **Inconsistency:**

- In maths students learn that data is either continuous, discrete (rather than discontinuous) or categorical. Data that is measured is continuous.
- In maths students do not manipulate data (which has negative connotations) – they process it

- In geography good questionnaire questions have tick boxes (closed) to reduce answer options. Make sure ranges for the tick boxes do not overlap, all options are covered, questions are set in a clear time frame (e.g. how many times do you go shopping per month, rather than just 'how many times do you go shopping?'), and are not be leading or biased. A survey should be anonymous in order to get more reliable outcomes.



## Guide to Maths for Geographers

- In a random sample, every member / item within a population has an equal chance of being selected.
- To select a random sample of size 10 you can:
  - give every population member a number
  - generate 10 random numbers and select the members with those numbers for the sample.
- For a small sample size you can put 'numbers into a hat' and draw them out at random. This is not practical for a large sample size.

- A sample that is too small can give biased results. This is particularly true for geographical fieldwork. Surveying two sites along a river, for example, would yield insufficient data and therefore would count as an unreliable sample. Similarly, measurement of a river's depth should take into account variability of the river bed. An uneven bed should have more samples than a regular cross-section.
- For data that is grouped by, for example, age bands, you can take a stratified sample, so the sample reflects the proportion of each age group in the population. This would be important in a fieldwork survey of a town to get a fair and reliable survey or sample of people's attitudes and opinions.

- You can use lines of best fit or follow the trend of a graph to estimate 'missing' data values. Estimating values that lie within the range of given values is called interpolation, though students may not learn this term. Estimating values that lie outside the range of given values is called extrapolation, though students may not learn this term. Interpolation is more likely to be accurate than extrapolation.